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In accordance with the suggestions contained in these letters, the Secretaries would ask authors of papers intended for publication by this Society to enclose with each paper a "condensed statement of the topics treated of." It will probably be some little time before a satisfactory uniform system is arrived at, and the Secretaries will welcome any suggestions which may conduce to this desirable end. For the present the particulars furnished by the authors themselves will generally be printed as they stand; or if any additions are made by the Editors these will be clearly indicated.

E. W. MAUNDER, H. H. TURNER, } *Secretaries.*

Note on Hansen's Lunar and Planetary Theories. By Professor Ernest W. Brown.

The integration of the equations for the mean anomaly and the radius vector in Hansen's lunar theory depend chiefly on the calculation of a certain function $\overline{W}$ which has the value *

$$\overline{W} = -1 - \frac{h_0}{h} + 2\frac{h}{h_0}\frac{\bar{r}}{a_0}\frac{1 + e\cos(\bar{f} + n_0 yt + \pi_0 - \chi)}{1 - e_0^2}.$$

In this expression h, e, χ refer to the instantaneous ellipse, and therefore, in disturbed motion, they are implicit functions of the time; h_0, n_0, y, π_0 are absolute constants referring to a certain auxiliary ellipse, of constant size and shape, situated in the plane of the instantaneous orbit, the mean anomaly of this ellipse being denoted by $n_0 z$. In undisturbed motion, $n_0 z$ is of the form, $n_0 t$ + *const.*; in disturbed motion, $n_0 z$ contains, in addition, terms depending on the action of the Sun, and it is therefore a function of the time. The symbols $\bar{r}$, $\bar{f}$ denote the radius vector and true anomaly of the point on the auxiliary ellipse where the latter is cut by the radius vector of

* *Darlegung der theoretischen Berechnung, &c., Abh. der K. Sächs. Ges. der Wiss.* vol. vi. p. 104.

the Moon; they are therefore functions of the one variable z and thence of the time.

It is necessary to insert, in the above expression for $\overline{\mathrm{W}}$, the disturbed values of h, e, χ; but instead of finding their values directly, Hansen differentiates W with respect to the time and then inserts the values of dh/dt, de/dt, $d\chi/dt$, thus reducing the three integrations to one. In performing the differentiation and subsequent integration of $\overline{\mathrm{W}}$ he has shown that we may consider $\bar{r}, \bar{f}$ as constants. The proofs of the theorems* by which this result is obtained are long and, as regards the particular function of the elements used by Hansen, unnecessary; the theorem can, in fact, be shown to be a simple application of the integral calculus.

The above expression for $\overline{\mathrm{W}}$ may be put into the form

$$\overline{\mathrm{W}} = \mathrm{L}_1 + \mathrm{L}_2\,\bar{r} + \mathrm{L}_3\,\bar{r}\cos\bar{f} + \mathrm{L}_4\,\bar{r}\sin\bar{f},$$

where L_1, L_2, L_3, L_4 contain t only through h, e, χ, $n_0 yt$. In the expressions for $\bar{r}, \bar{f}$ in terms of z (or of t), suppose that t be replaced by τ and z by ζ, and let $\bar{\rho}$, $\bar{\phi}$ denote the resulting expressions of $\bar{r}, \bar{f}$. Then, considering τ, and therefore ζ, as constant, we may write

$$\begin{aligned}\mathrm{W} &= \int\frac{d\mathrm{L}_1}{dt}dt + \bar{r}\int\frac{d\mathrm{L}_2}{dt}dt + \bar{r}\cos\bar{f}\int\frac{d\mathrm{L}_3}{dt}dt + \bar{r}\sin\bar{f}\int\frac{d\mathrm{L}_4}{dt}dt \\ &= \left[\int\left\{\frac{d\mathrm{L}_1}{dt} + \bar{\rho}\,\frac{d\mathrm{L}_2}{dt} + \bar{\rho}\cos\bar{\phi}\,\frac{d\mathrm{L}_3}{dt} + \bar{\rho}\sin\bar{\varphi}\,\frac{d\mathrm{L}_4}{dt}\right\}dt\right]_{\tau=t}\end{aligned}$$

If, then, we denote by W the value of $\overline{\mathrm{W}}$ when $\bar{\rho}$, $\bar{\varphi}$ replace $\bar{r}, \bar{f}$ in the expression first given, we have

$$\overline{\mathrm{W}} = \left[\int\frac{d\mathrm{W}}{dt}dt\right]_{\tau=t}.$$

That is to say, the expression for $\overline{\mathrm{W}}$ can be differentiated, the disturbed values of the elements substituted, and the resulting expression integrated, with $\bar{r}$, $\bar{f}$ constant during the whole process. The object of the introduction of τ, $\bar{\rho}$, $\bar{\varphi}$ is merely to prevent confusion between the quantities which remain variable and those which are considered constant.

The same result is available in the planetary theory,† but it is somewhat simplified by the absence of the term $n_0 yt$.

Haverford College, Pa.:
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* *Fundamenta nova, &c.*, pp. 22–25. *Commentatio de corporum cœlestium perturbationibus. Astr. Nach.* vol. xi. col. 322.

† *Auseinandersetzung einer zweckmässigen Methode, &c., Abh.* vol. v.